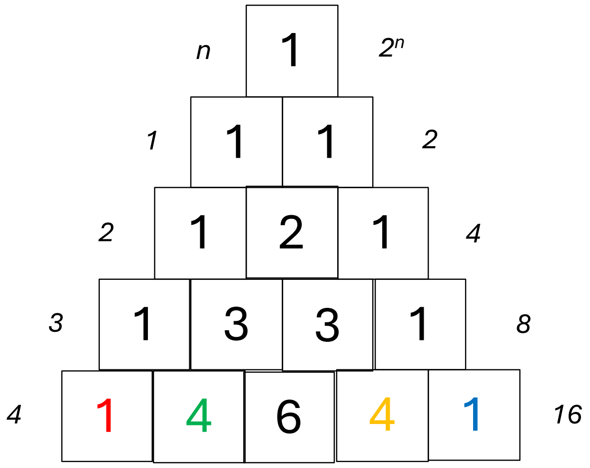
What are the odds?

In his lessons on card play last year, Paul Lavings described finesses as taking a bet on the placement of the cards. This year, Steve Geddes taught us how to use the bidding to help locate the cards to improve our odds with finesses and other parts of declarer play.

But sometimes you don't have much more information than the simple odds based on probability theory. This caused a discussion with my opponent when his play of the A and K of trumps successfully had the Q drop. When I noted the luck of this, he claimed there was a greater than 50% chance the cards would split 2-2 when he had a 6-3 holding (I don't recall the exact number). I disputed this, and after a while, my opponent agreed. I have to admit my opponent was right about the odds of the Q dropping, but not the odds on the 2-2 split.

Blaise Pascal developed probability theory through correspondence with Pierre de Fermat to answer questions about gambling posed to him by a friend (for the historically inclined, you can find out more about the story here https://www.ms.uky.edu/~dmu228/ma320/pascal_invention_probability.pdf)

Pascal's triangle (pictured below) is a convenient way for determining what mathematicians call *binomial coefficients.* For our purposes, it represents the ways a given number of cards *(n)* can be distributed between two hands. The triangle itself is built by creating an additional row of numbers offset from the previous row by half, putting a one on each end and otherwise filling each space with the sum of the numbers above it.



For our purposes, Pascal's Triangle can be read as the number of ways our Left Hand Opponent (LHO) can hold a certain number of cards. If declarer and dummy have nine cards in a suit, we are missing four cards in that suit. The last line is the case for opponents having four cards between them; it reads 1,4,6,4,1.

This tells us there is exactly one distribution where LHO holds zero cards (They are void and our RHO holds four cards), there are four distributions that they can hold one card as it can be any one of the four missing cards, there are six distribution that they hold two, four distributions that they hold three (missing any one of four cards), and only one distribution that they can hold all four.

We can see that from looking at the possible combinations if we are missing the Q432: The possible combinations of our opponents' two hands (LHO-RHO) are:

Q432-void, Q-432, 4-Q32, 3-Q42, 2-Q43 Q4-32, Q3-42, Q2-43, 43-Q2, 42-Q3, 32-Q4 432-Q, Q32-4, Q42-3, Q43-2, void-Q432.

We first assume that these distributions are equally likely. For example, this means that the probability of holding one card only is 4/16 or 25%.

The probability of the number of cards held by each opponent is therefore:

4-0	1/16 =	6.25%
3-1	4/16 =	25.00%
2-2	6/16 =	37.50%
1-3	4/16 =	25.00%
0-4	1/16 =	6.25

Now, let's narrow the question to whether the Q would drop in the first two tricks. We know it will when the cards split 2-2 (6 chances out of 16), and it will also in each case where one of my opponents has a singleton and it is the Q (two lots of one chance in 16). Therefore, the total cases where the Q will drop are 6+1+1, which, divided by 16, gives us 50%. On the raw probabilities from Pascal's triangle without any additional knowledge from the bidding, the odds are even that the Q will drop.

A rather dense book *The Mathematical Theory of Bridge* by Émile Borel and André Chéron, takes the analysis a stage further. Their calculations reflect the fact that all the cards have to be distributed between four hands, and this creates a slight change in what they call the 'division of a residue between two hidden hands.' For the case of 4 cards, they give the probabilities (Table 51) as:

3-1 or 1-3	49.739%
2-2	40.696%
4-0 or 0-4	9.565%

These probabilities only apply before ANY card has been played by either of the unseen hands. We can see that by realising that one combination, that the hand playing the card had a void, is immediately ruled out if they play a suit in that card.

This distribution does not depend on how the nine other cards in that suit are distributed between declarer and dummy. Later, they give a table (Table 52) of the probabilities for the placement of a given card in one of the hands among a residue of cards – in this case, our Q in a residue of 4. But we can calculate them in this case from the first table.

The hand holding the Q will have it

Void	0%	
Singleton	12.43%	(49.739 /4)
Doubleton	40.70%	(split 2-2)
One of three	37.30%	(49.739 *3/4)
One of four	9.57%	

That is, when declarer and dummy have a nine-card suit (no matter how distributed) headed by the AK but missing the Q, the odds that the Q will drop on the first two tricks are 12.43% + 40.70% = 53.13%.

So, hence the phrase 'eight ever, nine never' about trying to finesse the Q when you have the A and K, unless, of course, you have additional information from the bidding.

I will put a copy of the complete Tables on the documents part of the website for any interested members.

In the next issue, a short look at how information from the play can change the probabilities.

One bridge hand can be composed 635,013,359,600 different ways.

A whole deal of four hands can be composed in 53,644,737,765,488,792,839,237,440,000 different ways.

Table 51. Division of a residue between two hidden hands.

A Residue of	Will be divided between two hidden hands	Probability %
2 cards	1-1	52
	2-0 or 0-2	48
3 cards	2-1 or 1-2	78
	3-0 or 0-3	22
4 cards	3-1 or 1-3	49.7391
	2-2	40.6957
	4-0 or 0-4	9.5652
5 cards	3-2 or 2-3	67.8261
	4-1 or 1-4	28.2609
	5-0 or 0-5	3.9130
6 cards	4-2 or 2-4	48.4472
	3-3	35.5280
	5-1 or 1-5	14.5342
	6-0 or 0-6	1.4907
7 cards	4-3 or 3-4	62.1739
	5-2 or 2-5	30.5217
	6-1 or 1 -6	6.7826
	7-0 or 0-7	0.5217
8 cards	5-3 or 3-5	47.1213
	4-4	32.7231
	6-2 or 2-6	17.1350
	7-1 or 1-7	2.8558
	8-0 or 0-8	0.1648
9 cards	5-4 or 4-5	58.9016
	6-3 or 3-6	31.4142
	7-2 or 2-7	8.5675
	8-1 or 1-8	1.0709
	9-0 or 0-9	0.0458

A Residue of	Will be divided between two hidden hands	Probability %
10 cards	6-4 or 4-6	46.1973
	5-5	31.1832
	7-3 or 3-7	18.4789
	8-2 or 2-8	3.7798
	9-1 or 1-9	0.3500
	10-0 or 0-10	0.0108
11 cards	6-5 or 6-6	57.1692
	7-4 or 4-7	31.7607
	8-3 or 3-8	9.5282
	9-2 or 2-9	1.4437
	10-1 or 1-10	8.0962
	11-0 or0-ll	0.0020
12 cards	7-5 or 5-7	45.7354
	6-6	30.4902
	8-4 or 4-8	19.0564
	9-3 or 3-9	4.2348
	10.2 or 2-10	0.4620
	11-1 or 1-11	0.0210
	12-0 or 0-12	0.00027
13 cards	7-6 or 6-7	56.6247
	8-5 or 5-8	31.8514
	9-4 or 4-9	9.8307
	10-3 or 3-10	1.5729
	11-2 or 2-11	0.1170
	11-2 or 2-11	0.1170
	12-1 or 1-12	0.0032
	13.0 or 0-13	0.000019

Table 52. Length of suit in a hand with a given card

1st Column: Number of cards which the two hidden hands together have in the same suit
2 Column: A specified missing card in that suit held by a hand with N cards.
3 Column: Probability %,

Cards	N	%
2	1	52
	2	48
3	2	52
	1	26
	3	22
4	2	40.70
	3	37.30
	1	12.43
	4	9.57
5	3	40.70
	2	27.13
	4	22.61
	1	5.65
	5	3.91
6	3	35.53
	4	32.30
	2	16.15
	5	12.11
	1	2.42
	6	1.49
7	4	35.53
	3	26.65
	5	21.80
	2	8.72
	6	5.81
	1	0.97
	7	0.52

Cards	N	%
8	4	32.72
	5	29.45
	3	17.67
	6	12.85
	2	4.28
	7	2.50
	1	0.36
	8	0.16
9	5	32.72
	4	26.18
	6	20.94
	3	10.47
	7	6.66
	2	1.90
	8	0.95
	1	0.12
	9	0.046
10	5	31.18
	6	27.72
	4	18.48
	7	12.94
	3	5.54
	8	3.02
	2	0.76
	9	0.32
	1	0.035
	10	0.011

Cards	N	%
11	6	31.18
	5	25.99
	7	20.21
	4	11.55
	3	2.6
	9	1.18
	2	0.26
	10 or 1	0.087
	11	0.002
12	6	30.49
	7	26.68
	5	19.06
	8	12.7
	4	6.35
	9	3.18
	3	1.06
	10	0.39
	2	0.077
	11	0.019
	1	0.0017
	12	0.0003
13	7	30.49
	6	26.13
	8	19.6
	5	12.25
	9	6.81
	4	3.02
	10	1.21
	3	0.36
	11	0.099
	2	0.018
	12	0.003
	1	0.0002
	13	0.00002